A method for calculating the variance and confidence intervals for biomass estimates obtained from allometric equations

Alecia Nickless, Bob Scholes, Sally Archibald

Ecosystem Processes and Dynamics, Natural Resources and the Environment, CSIR, South Africa

ANickless@csir.co.za

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Why do we need to know how much biomass there is?

Standard format for woody biomass equation

$$y = ax^b \varepsilon$$

where y is biomass and x is the tree parameter (e.g. stem diameter)

$$\ln(y) = \ln(a) + b\ln(x) + \varepsilon^*$$

where ε^* is the error in the estimation of $\ln(y)$

~ simple linear regression theory can be used

The variance of a predicted value obtained from a new x value

$$E(\hat{\ln}(y_i)) = \hat{\mu}_i^* = \hat{\beta}_0^* + \hat{\beta}_1^* \ln(x_i)$$

$$\mathrm{Var}(\hat{\ln}(y_i)) = \hat{\sigma}_i^{2*} = \mathrm{MSE}(1 + \ln(x_i)(X'X)^{-1}\ln(x_i))$$

Additional regression parameters required for confidence interval calculation

The MSE from the original regression is required and

$X'X$ can be simplified to

$$\begin{pmatrix} n & \sum \ln x_j \\ \sum \ln x_j & \sum (\ln x_j)^2 \end{pmatrix}$$

Converting the logged biomass estimates into biomass

$$y_i \sim Log\ Normal$$

$$\hat{\mu}_i^* = \widehat{\ln(y_i)}$$

$$\hat{y}_i = \exp(\hat{\mu}_i^* + \hat{\sigma}^{2*}/2)$$

$$\hat{\sigma}^2 = \exp(2\hat{\mu}_i^* + 2\hat{\sigma}_i^{2*}) - \exp(2\hat{\mu}_i^* + \hat{\sigma}_i^{2*})$$

The biomass is LOG NORMALLY distributed…
The logged biomass is NORMALLY distributed

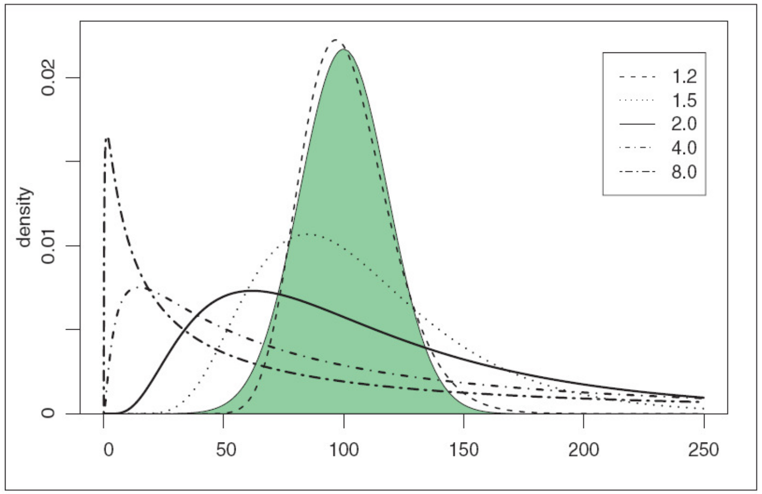
Asymmetrical confidence intervals for a Log Normal variable

$$\text{Lower Limit} = \hat{y}_i \exp[-(z^2_{1-\alpha/2}\hat{\sigma}_i^2 + \{\hat{\sigma}_i^2/2\}^2)^{1/2}]$$

$$\text{Upper Limit} = \hat{y}_i \exp[(z^2_{1-\alpha/2}\hat{\sigma}_i^2 + \{\hat{\sigma}_i^2/2\}^2)^{1/2}]$$

e.g. $z_{1-\alpha/2} = 1.96$

for the 95% confidence interval



Limpert *et al.*, 2001

Examples of woody biomass equations obtain from two different data sources

Woody Biomass: $\ln(\hat{y}_{Wi}) = \hat{\beta}^*_{W0} + \hat{\beta}^*_{W1} \ln(x_i)$

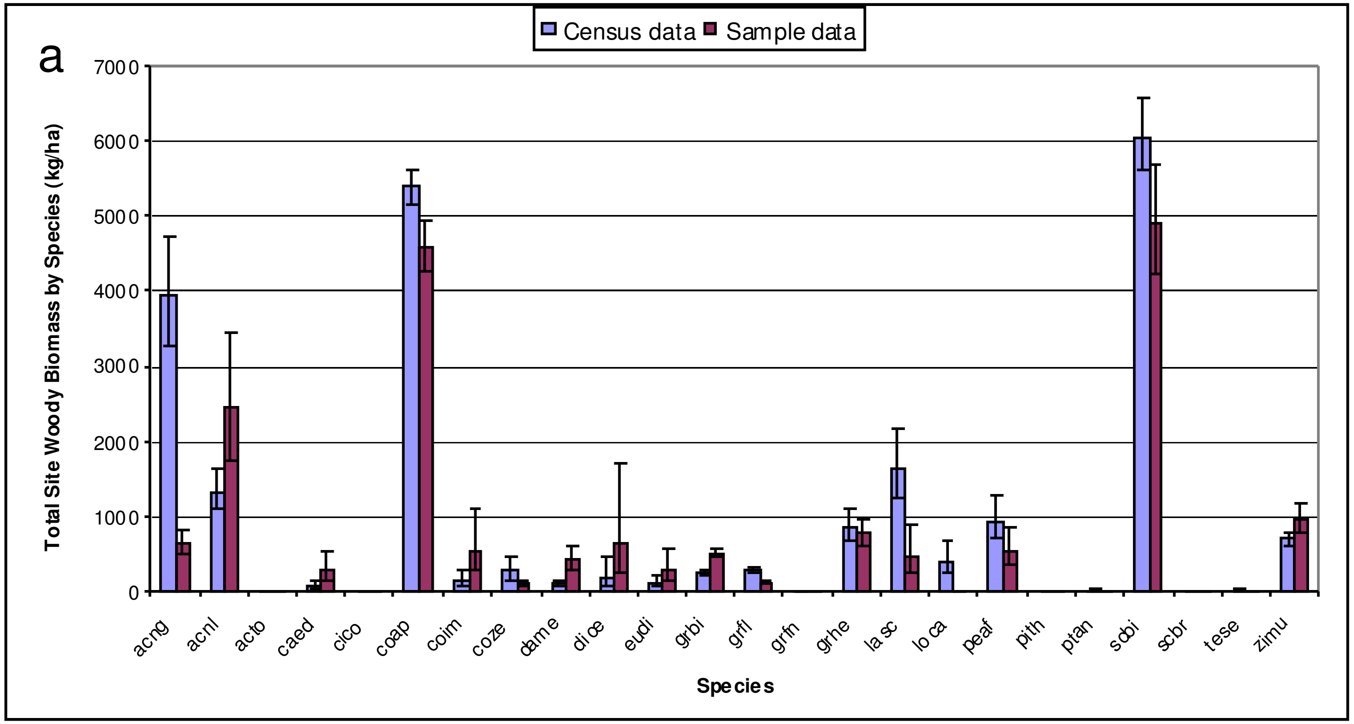
Source = Scholes 1988

Species	$\hat{\beta}^*_{W0}$	$\hat{\beta}^*_{W1}$	MSE	N	$\sum(\ln x_j)$	$\sum(\ln x_j)^2$	R^2	Range of DBH (cm)
Colophospermum mopane	-2.77	2.49	6.9×10^{-2}	30	56.18	112.56	0.96	1.8 – 23.0
Combretum apiculatum	-3.27	2.80	4.24×10^{-2}	30	61.37	133.39	0.98	2.1 – 18.2
Sclerocarya birrea	-3.35	2.62	3.67×10^{-2}	30	64.50	148.85	0.99	3.6 – 33.0

Source = Goodman 1990

Species	$\hat{\beta}^*_{W0}$	$\hat{\beta}^*_{W1}$	MSE	N	$\sum(\ln x_j)$	$\sum(\ln x_j)^2$	R^2	Range of DBH (cm)
Acacia grandicornuta	-3.31	3.07	1.19×10^{-1}	38	24.28	27.65	0.96	0.6 – 8.1
Acacia luederitzii	-3.57	3.35	1.38×10^{-1}	36	27.23	30.96	0.96	0.7 – 8.3
Acacia nigrescens	-3.55	3.06	5.92×10^{-2}	31	19.87	27.97	0.99	0.5 – 9.7
Acaica nilotica	-3.78	3.16	5.15×10^{-2}	38	27.17	32.16	0.99	0.8 – 8.3

Results from application to data from Skukuza flux tower



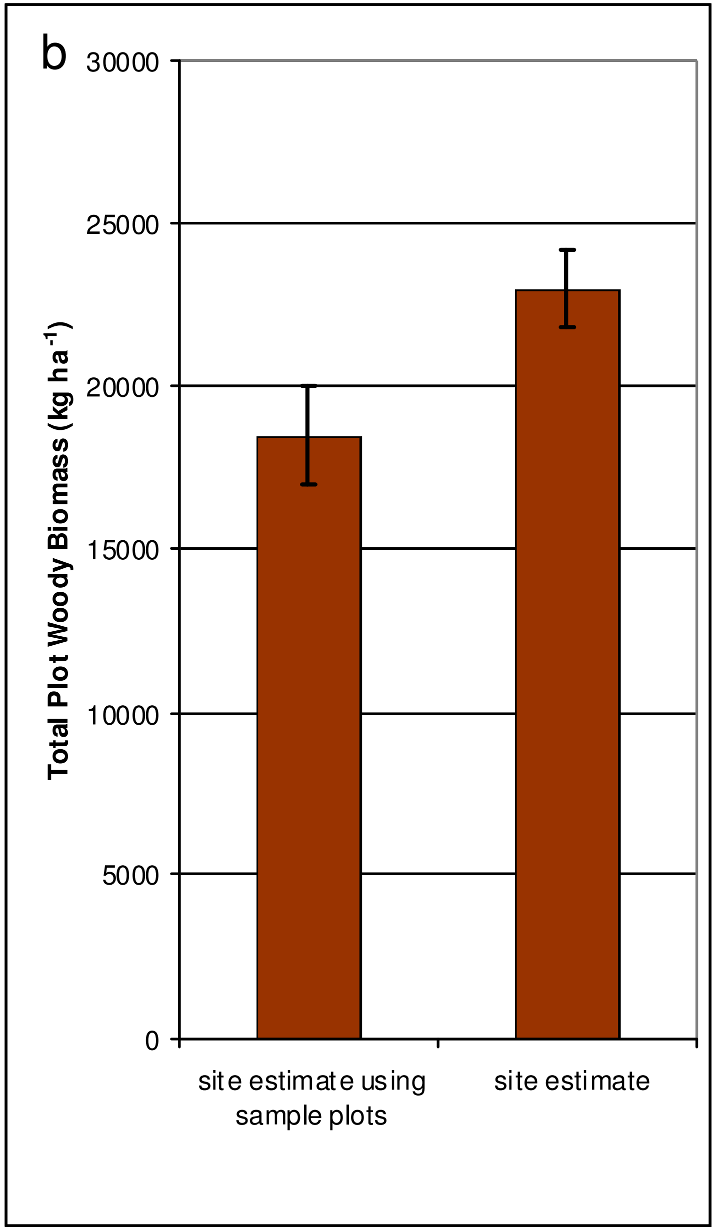
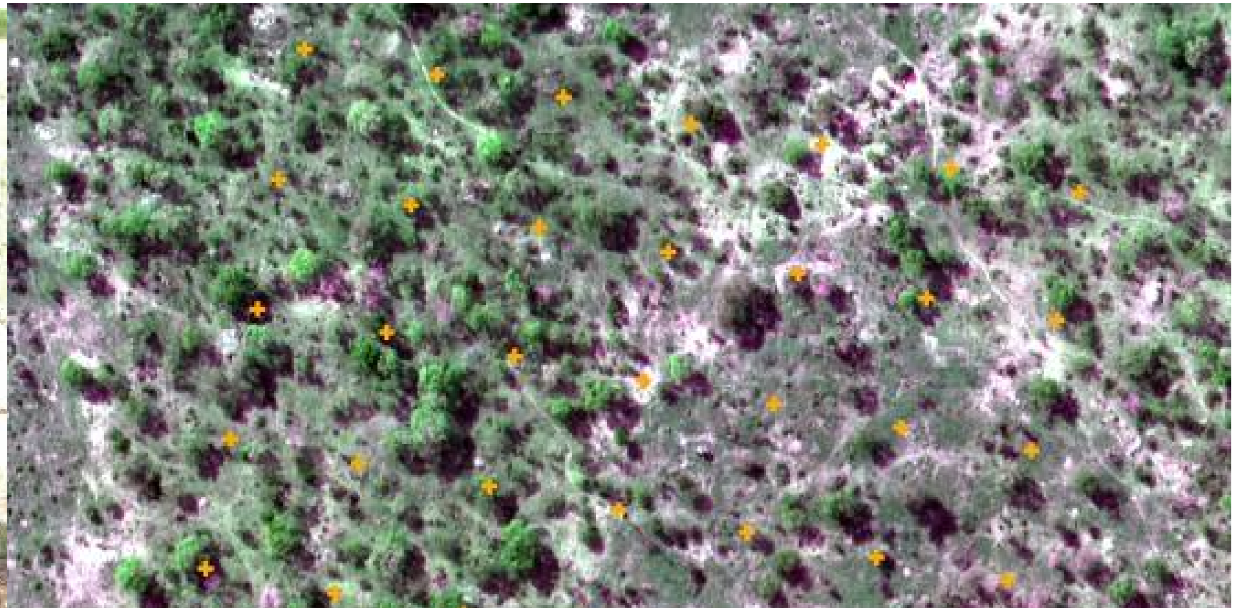


Fig. 1 Plot of total woody biomass per species (a) and for the entire 200 m by 200 m site (b) at the Skukuza flux site. In plot (a) bars on the left were obtained from a census of stem diameters taken at the site, and those on the right were obtained from a sample of stems.

Allometric relationships are required for calibration and validation



anickless@csir.co.za OR bscholes@csir.co.za

for more information

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